Regression:

Predicting House Prices

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Predicting house prices

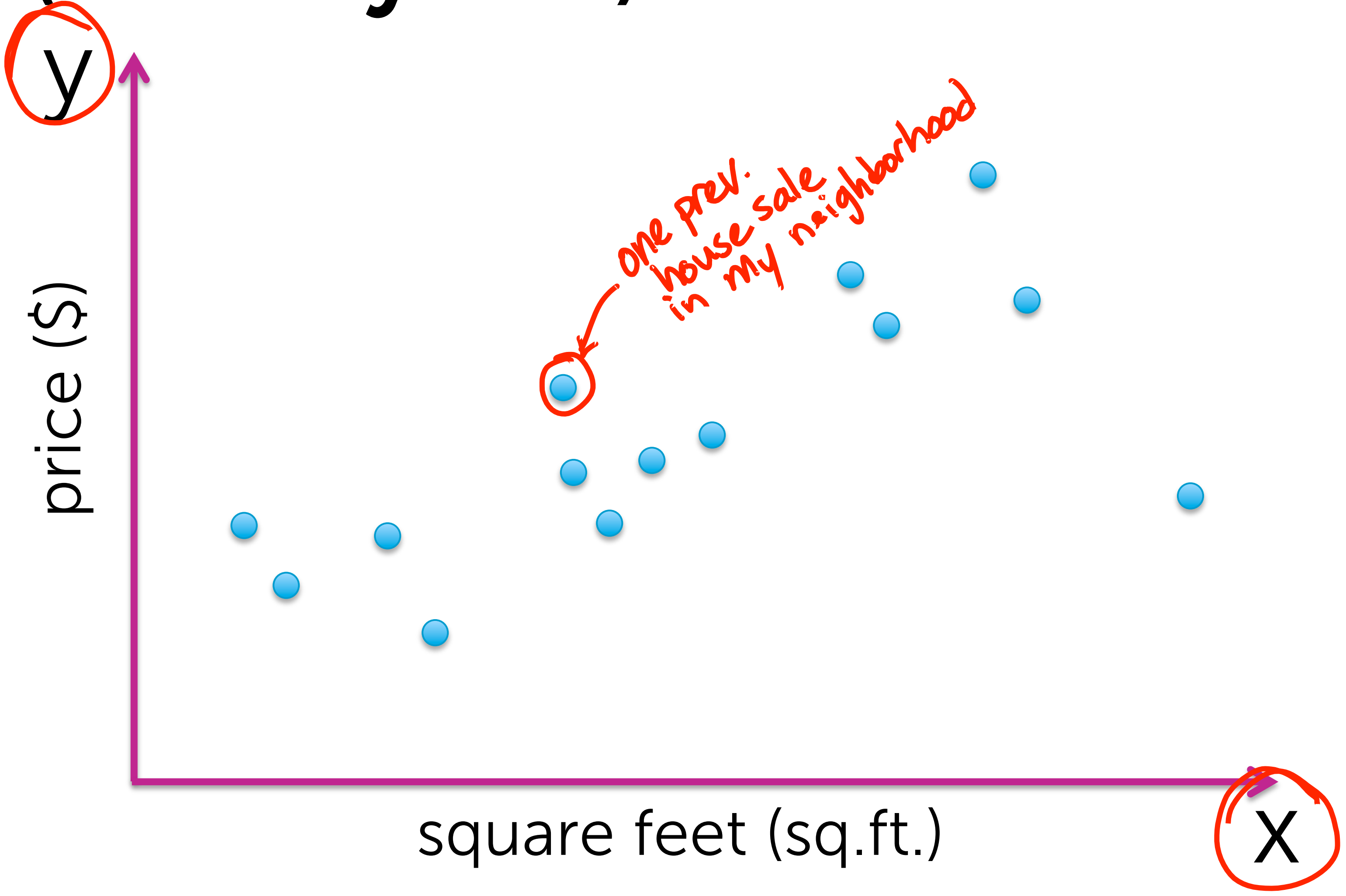
How much is my house worth?

How much is my house worth?

Look at recent sales in my neighborhood

- How much did they sell for?

Plot recent house sales
(Past 2 years)

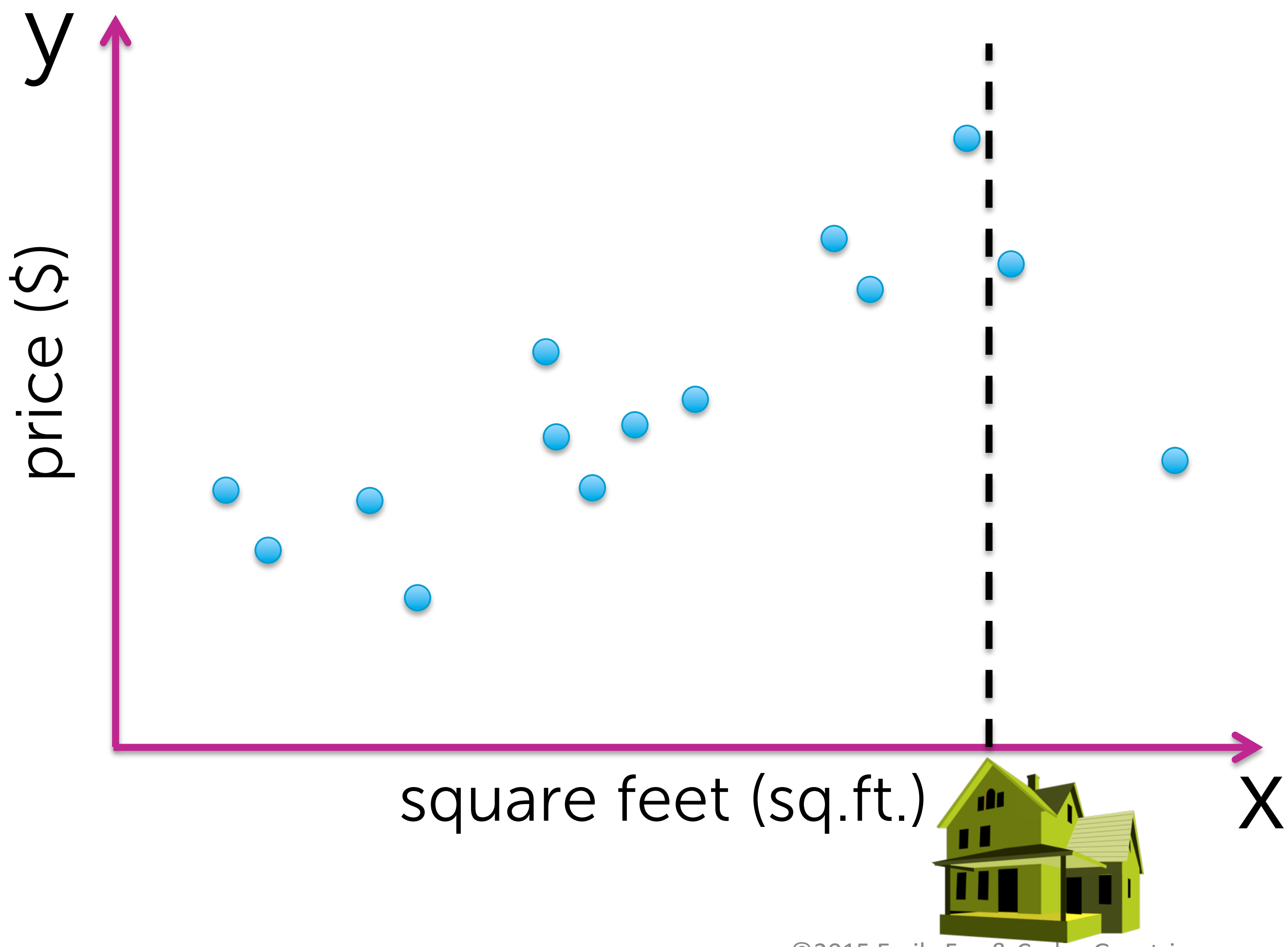


Terminology:

x – feature, covariate, or predictor

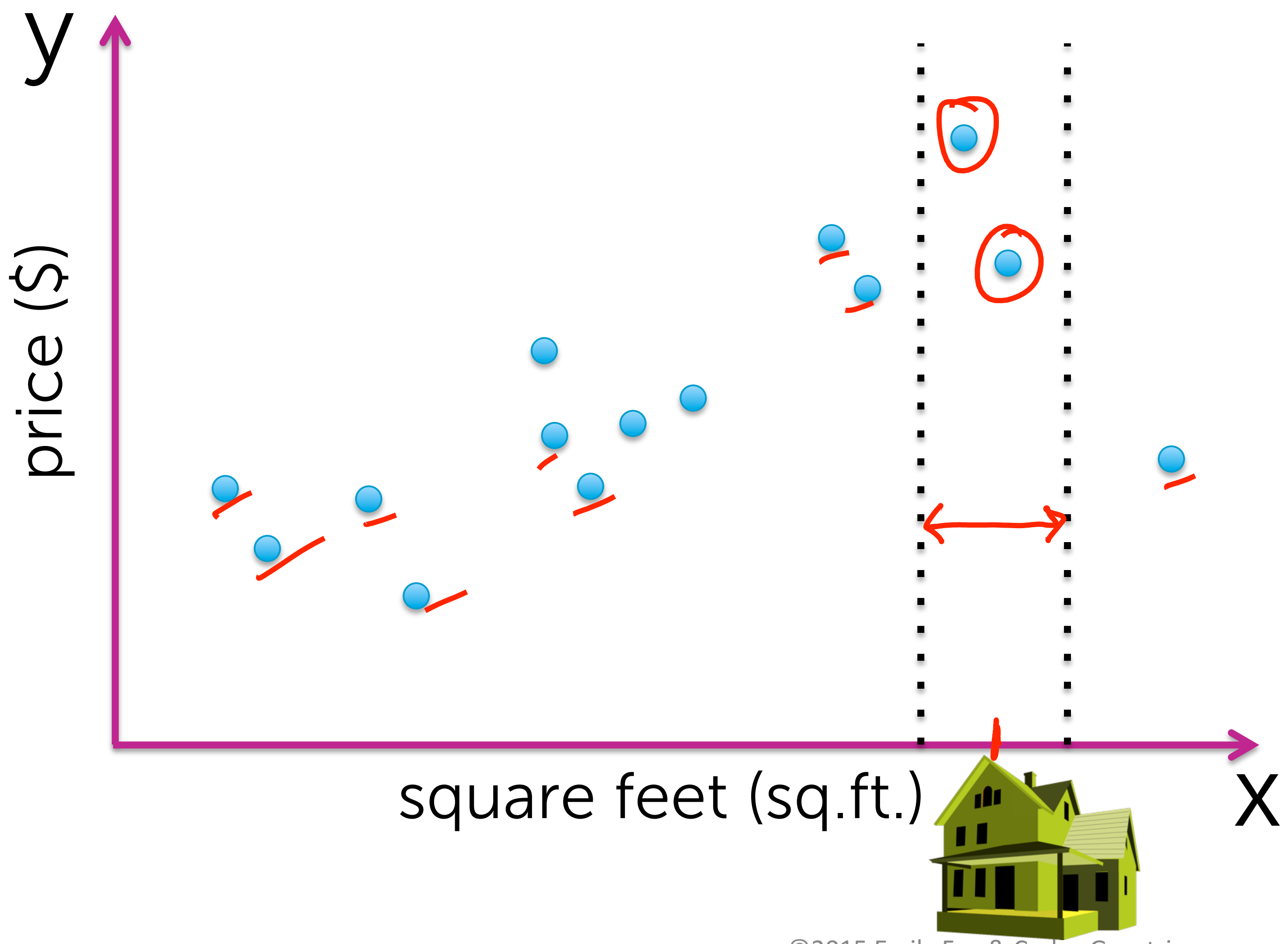
y – observation or response

Predict your house by similar houses



No house sold recently had *exactly* the same sq.ft.

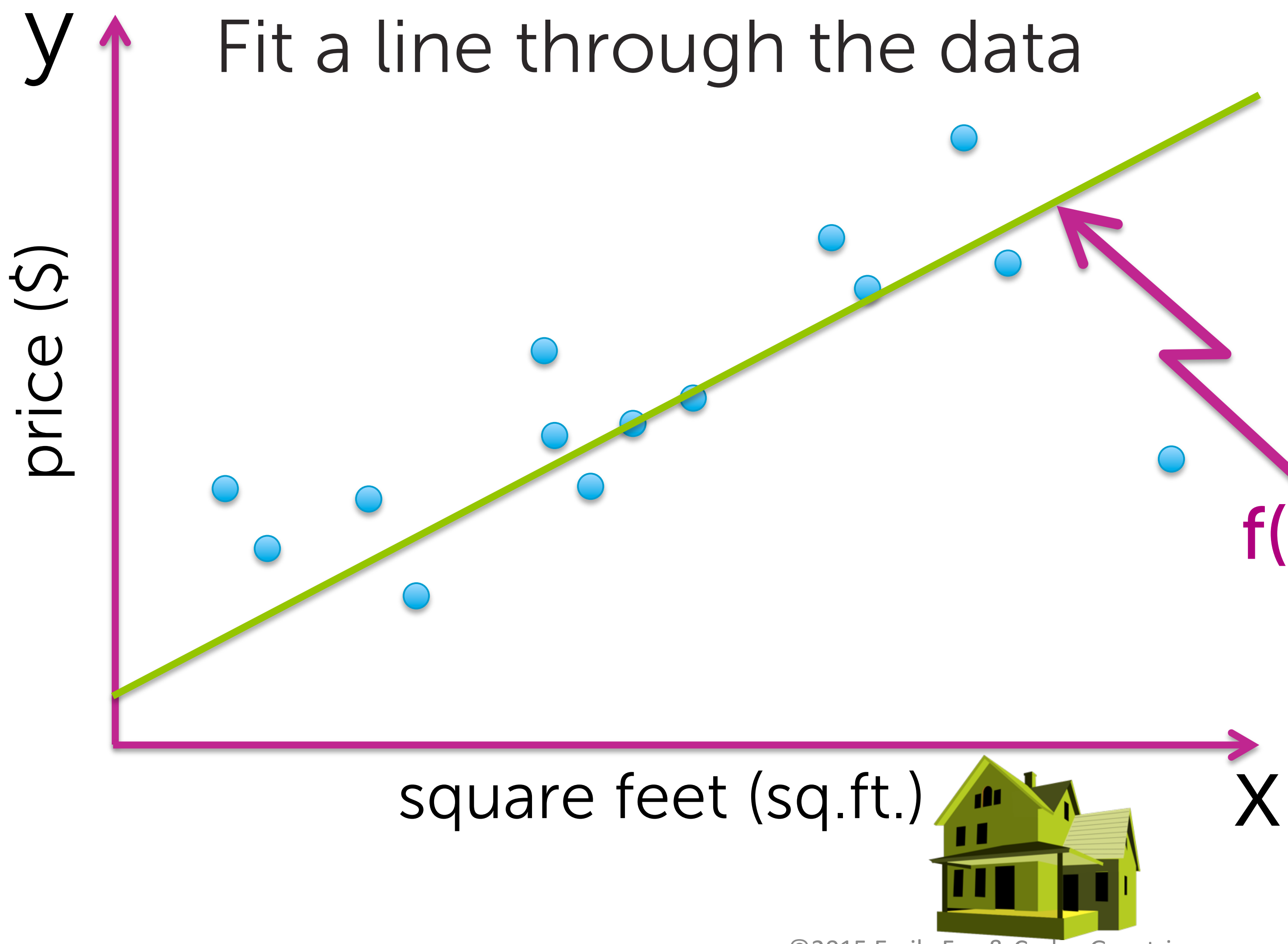
Predict your house by similar houses



- Look at average price in range
- **Still only 2 houses!**
- Throwing out info from all other sales

Linear regression

Use a **linear** regression model

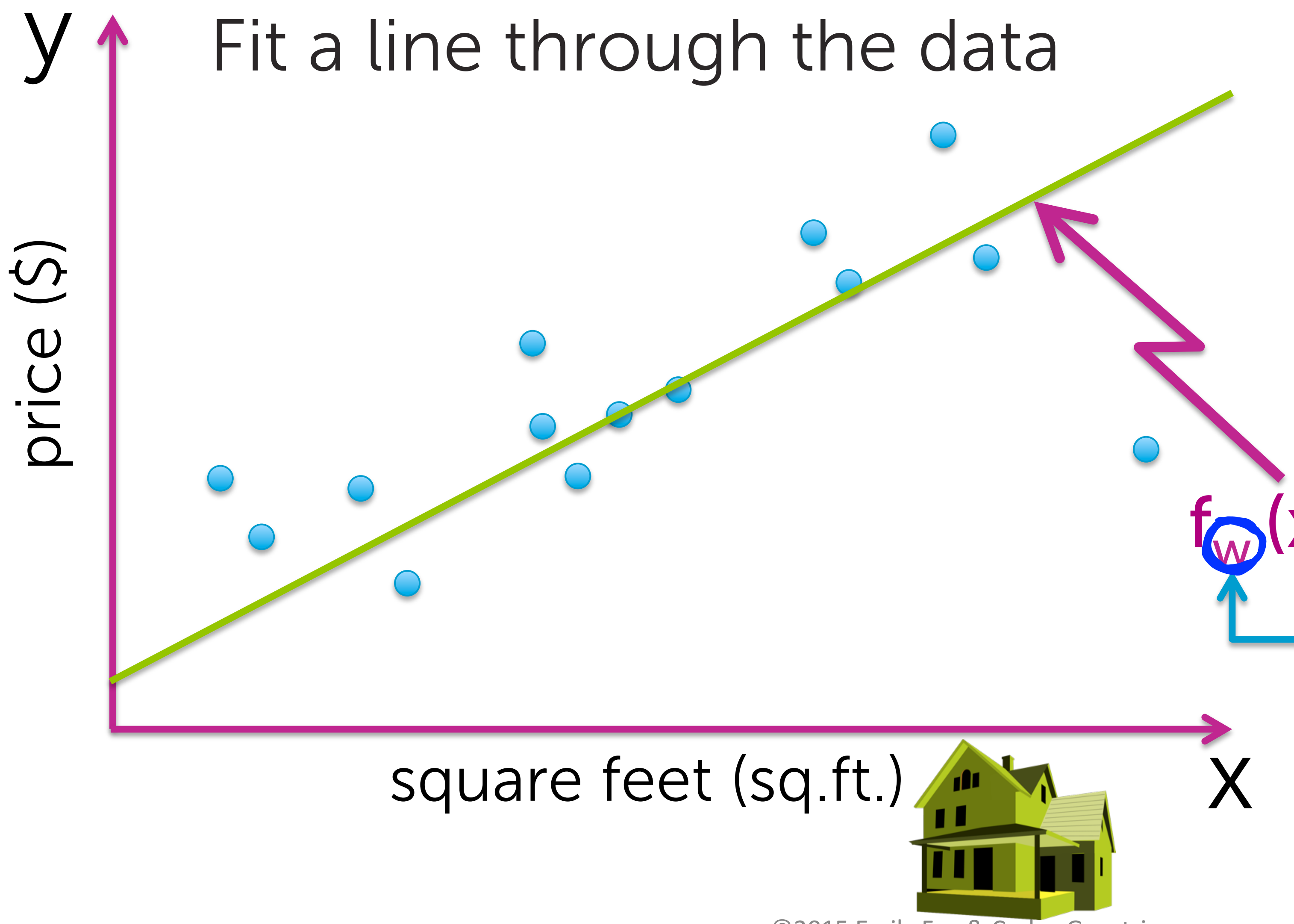


Fit a line through the data

$$f(x) = \underset{\text{intercept}}{w_0} + \underset{\text{slope}}{w_1}\, x$$

parameters
of model

Use a **linear** regression model

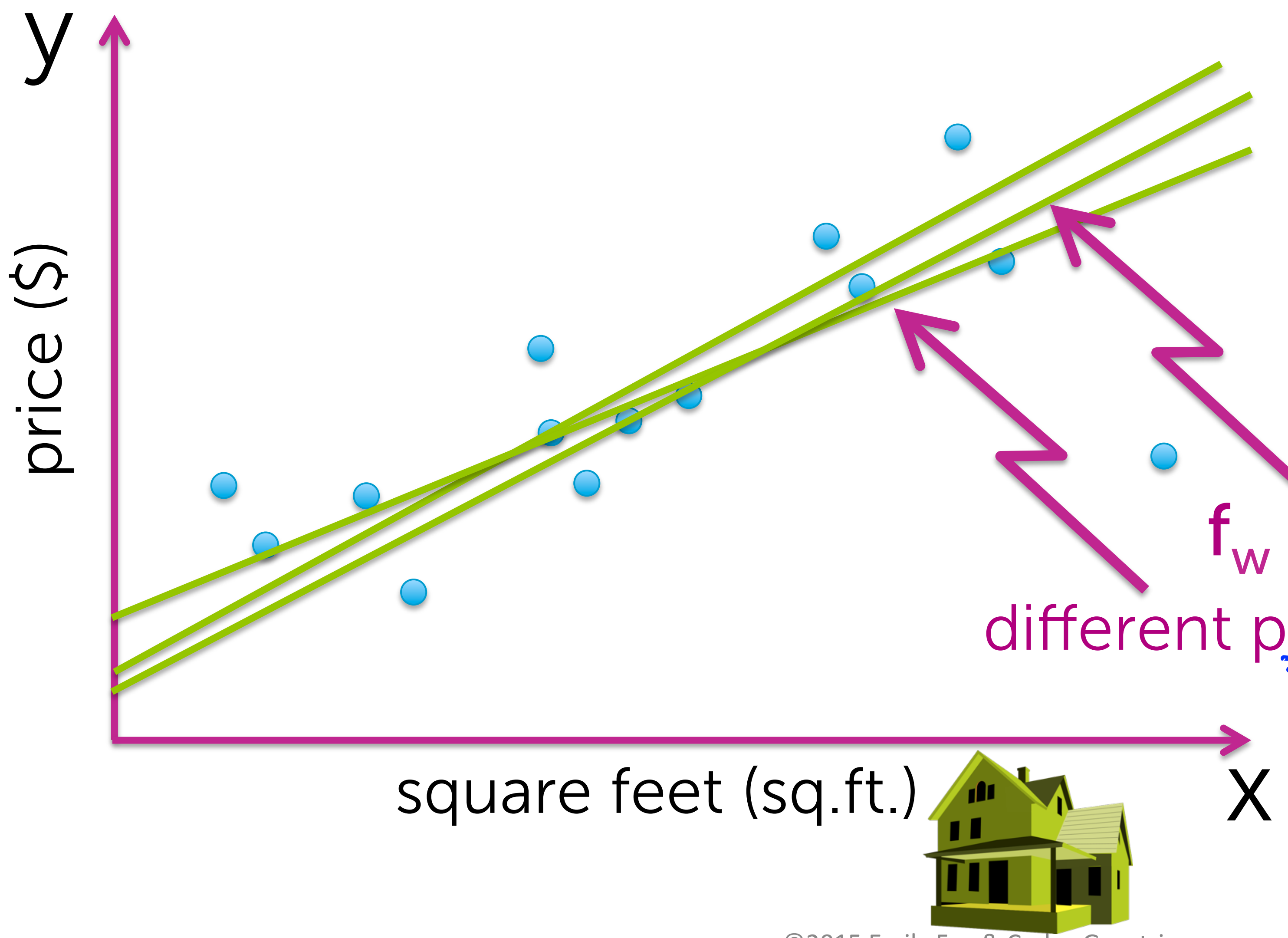


Fit a line through the data

$f_{w}(x) = w_0+w_1\,x$

function parameterized by $\boxed{w = (w_0, w_1)}$

Which line?

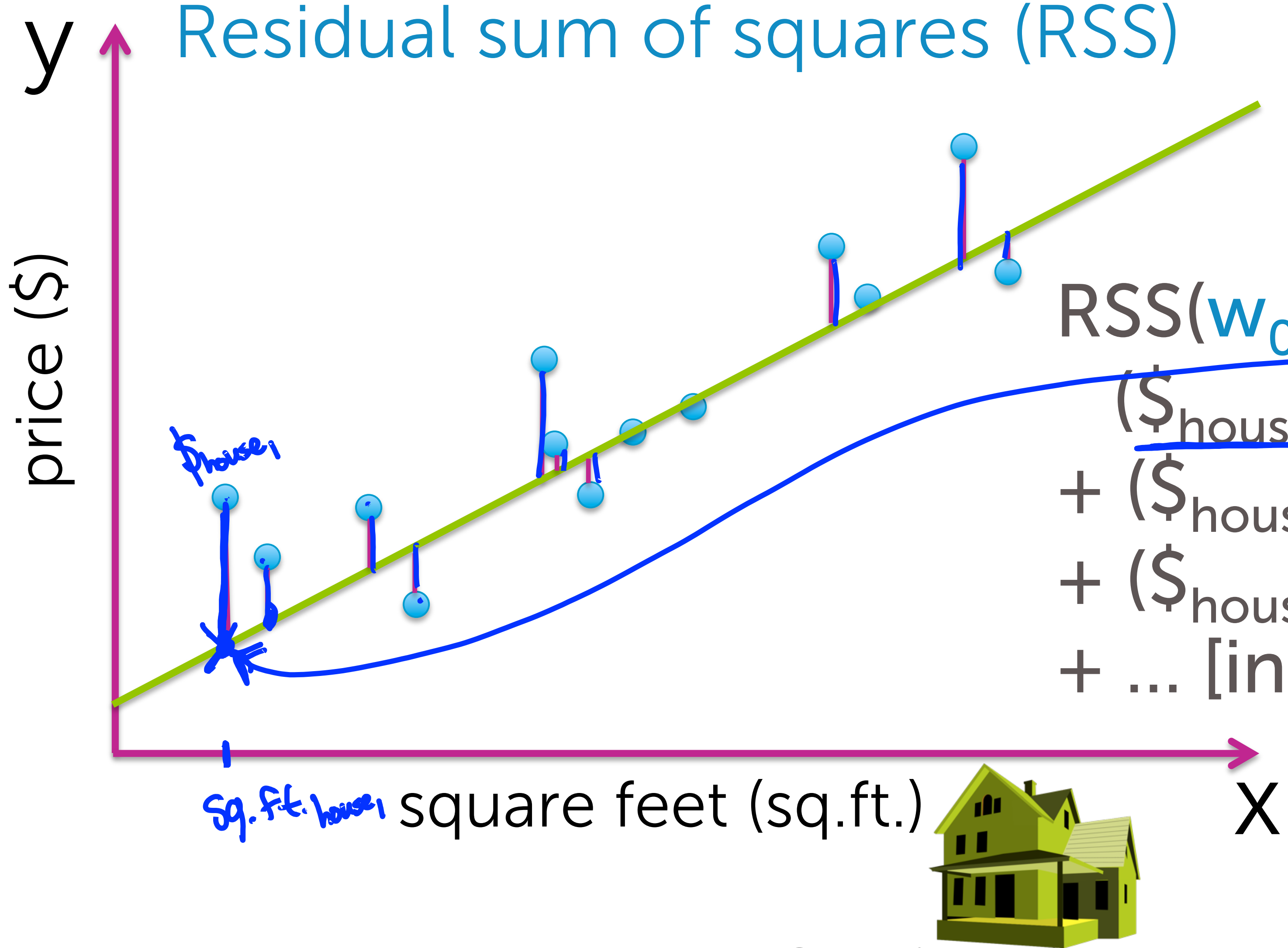


$f_w(x) = w_0+w_1 x$

different parameters w

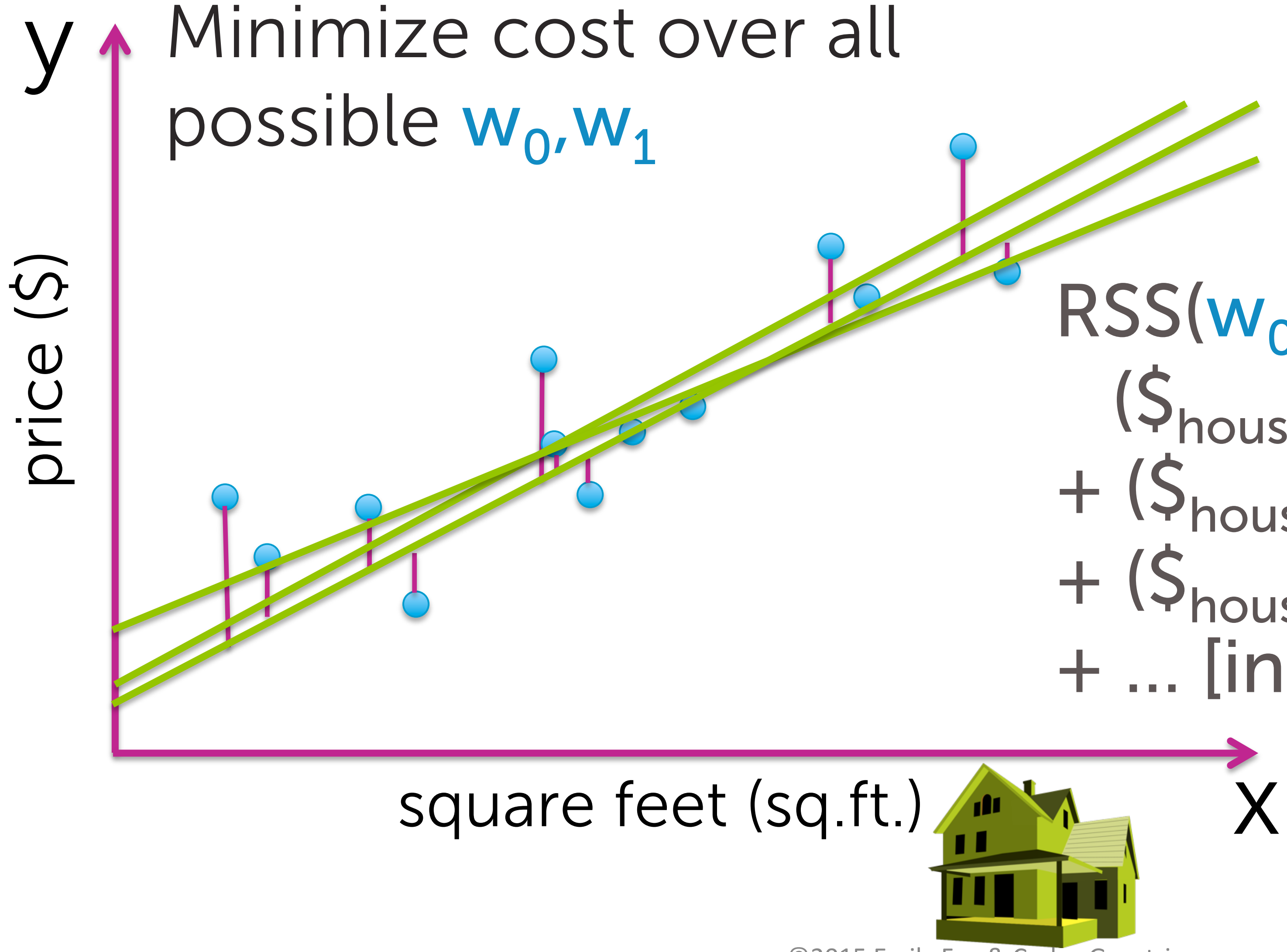
“Cost” of using a given line

Residual sum of squares (RSS)



$$
\begin{aligned}
\mathrm{RSS}(w_0,w_1) = \\
&(\underline{\$_{\text{house }1}}-[\boxed{w_0+w_1\text{sq.ft.}_{\text{house }1}}])^2 \\
+\ &(\$_{\text{house }2}-[w_0+w_1\text{sq.ft.}_{\text{house }2}])^2 \\
+\ &(\$_{\text{house }3}-[w_0+w_1\text{sq.ft.}_{\text{house }3}])^2 \\
+\ &\ldots\ [\text{include all houses}]
\end{aligned}
$$

Find "best" line



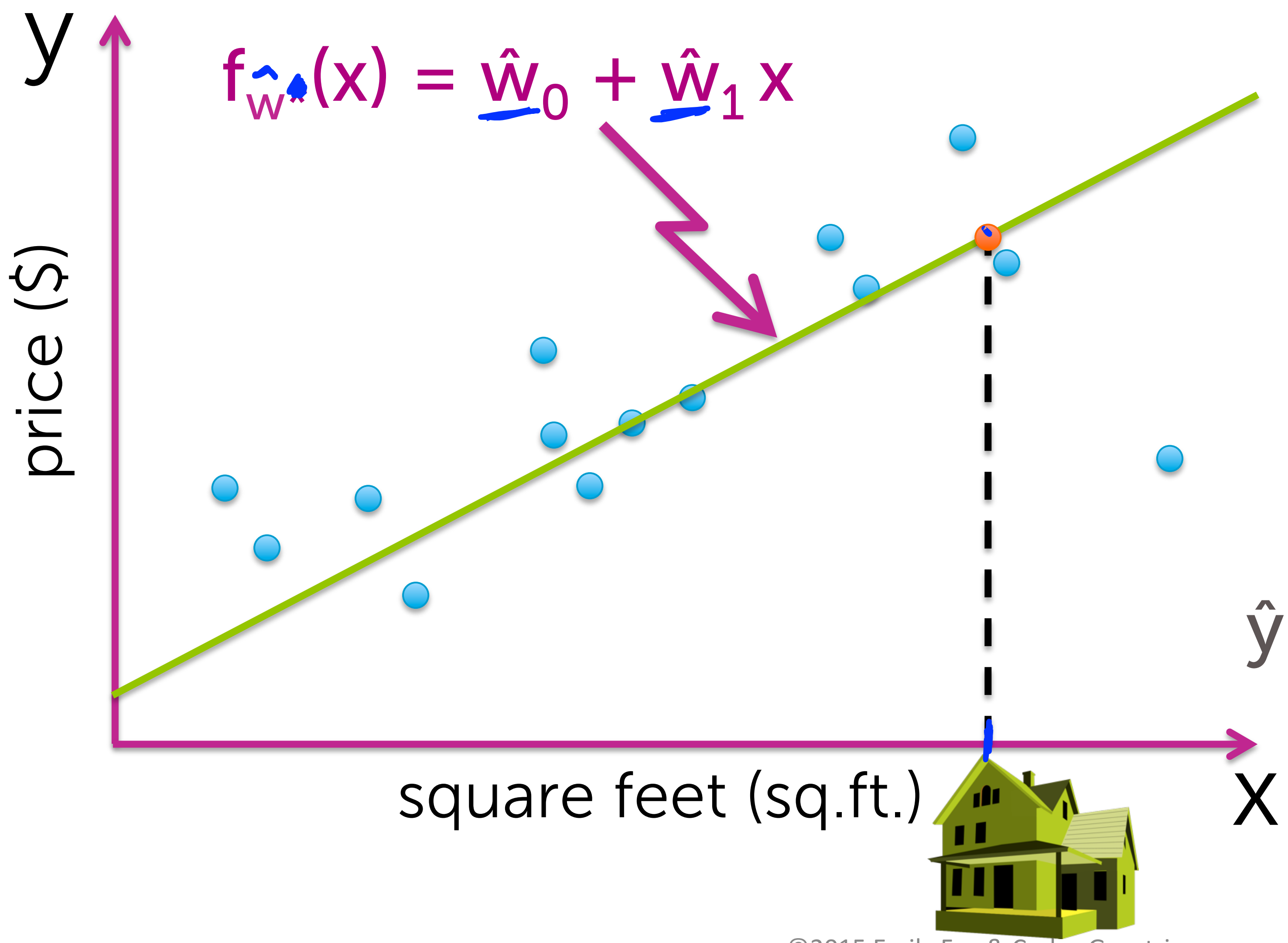
Minimize cost over all possible w_0, w_1

$$
\begin{aligned}
\mathrm{RSS}(w_0,w_1) = {} & (\$_{\text{house }1} - [w_0 + w_1 \text{sq.ft.}_{\text{house }1}])^2 \\
& + (\$_{\text{house }2} - [w_0 + w_1 \text{sq.ft.}_{\text{house }2}])^2 \\
& + (\$_{\text{house }3} - [w_0 + w_1 \text{sq.ft.}_{\text{house }3}])^2 \\
& + \ldots \text{ [include all houses]}
\end{aligned}
$$

$$\downarrow$$

$$\boxed{\hat{W} = (\hat{w}_0, \hat{w}_1)}$$

Predicting your house price

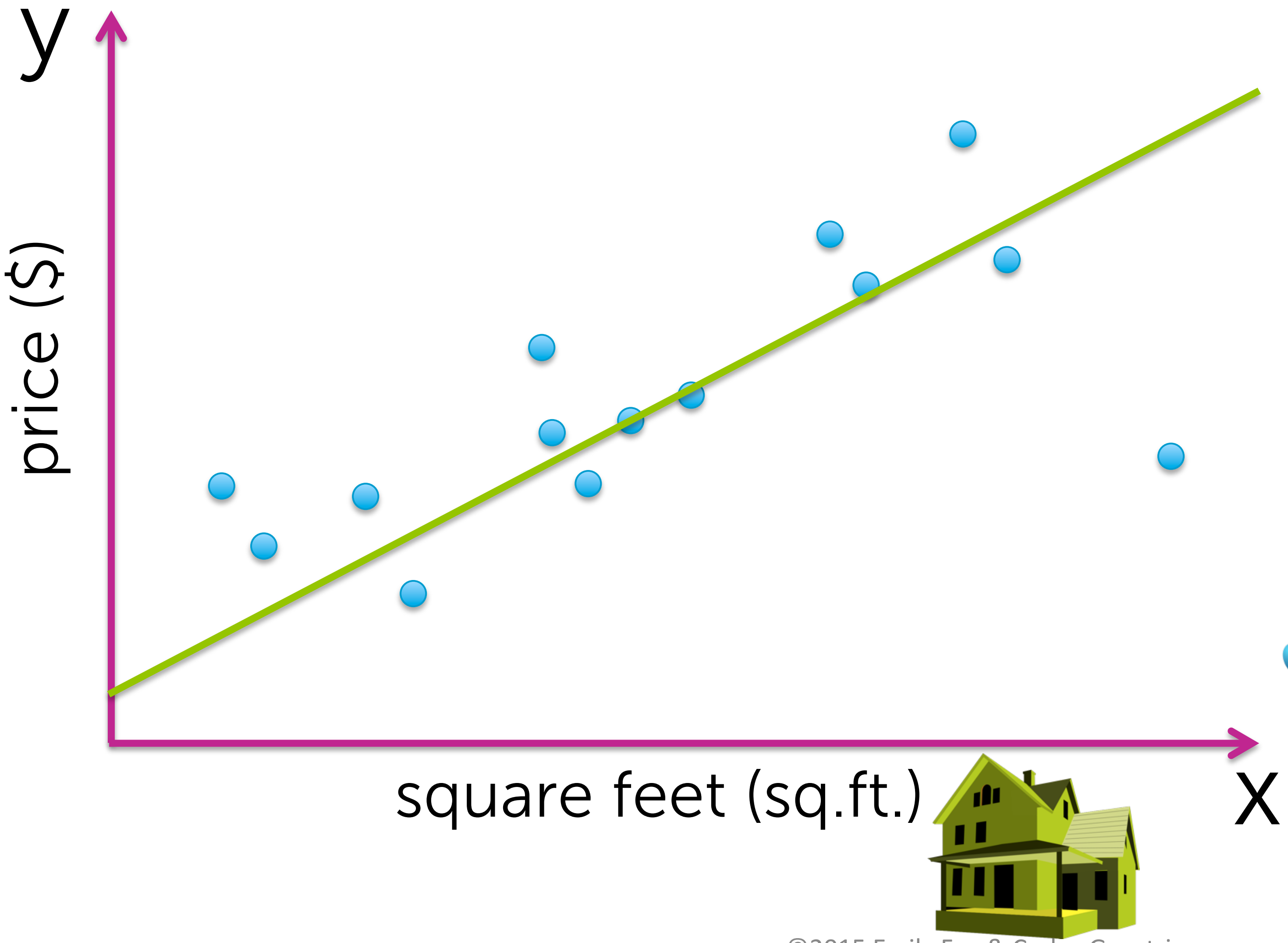


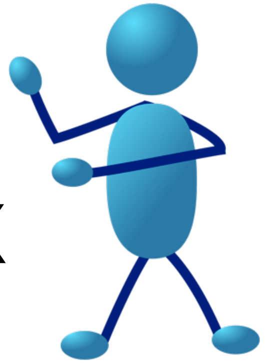
Best guess of your house price:

$$\hat{y} = \hat{w}_0 + \hat{w}_1 \,\text{sq.ft.}_{\text{your house}}$$

Adding higher order effects

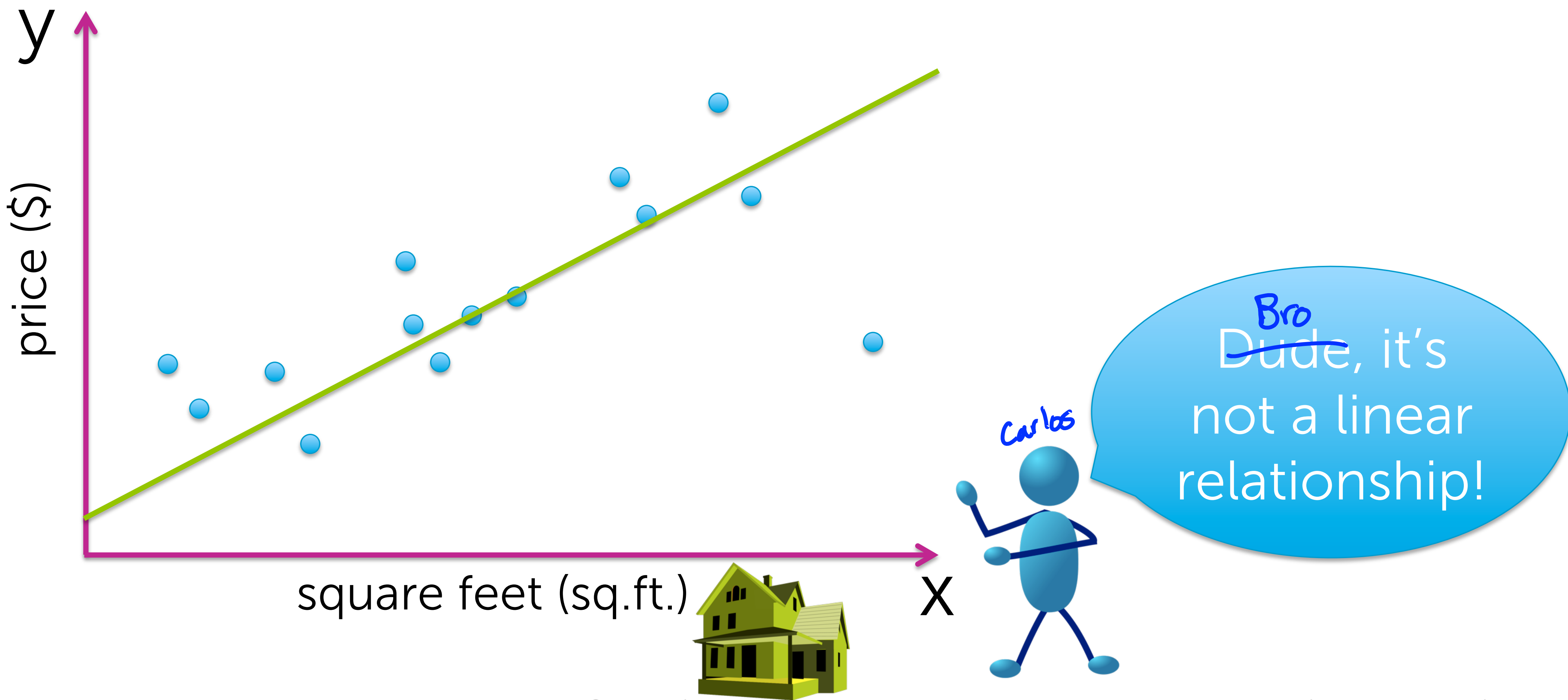
Fit data with a line or … ?



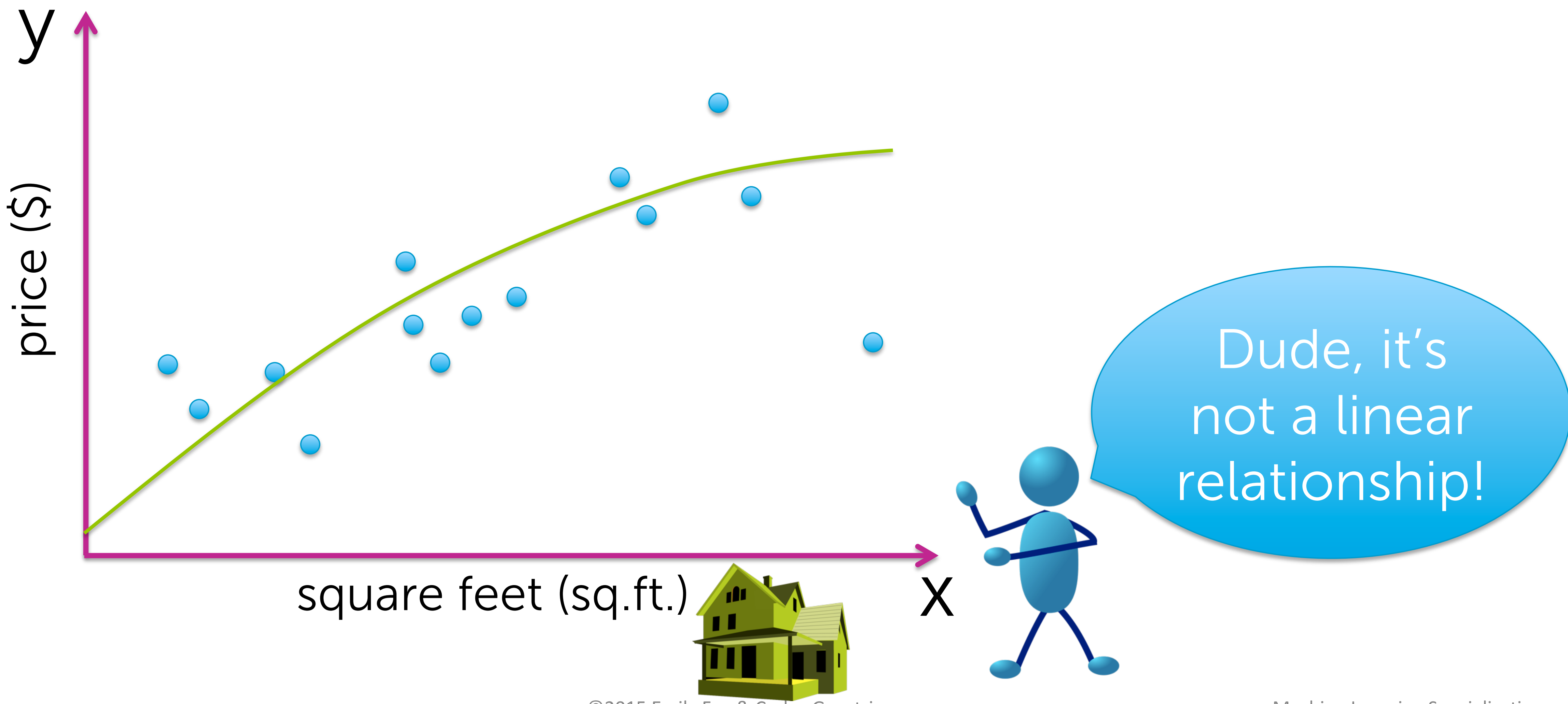


You show
your friend
your analysis

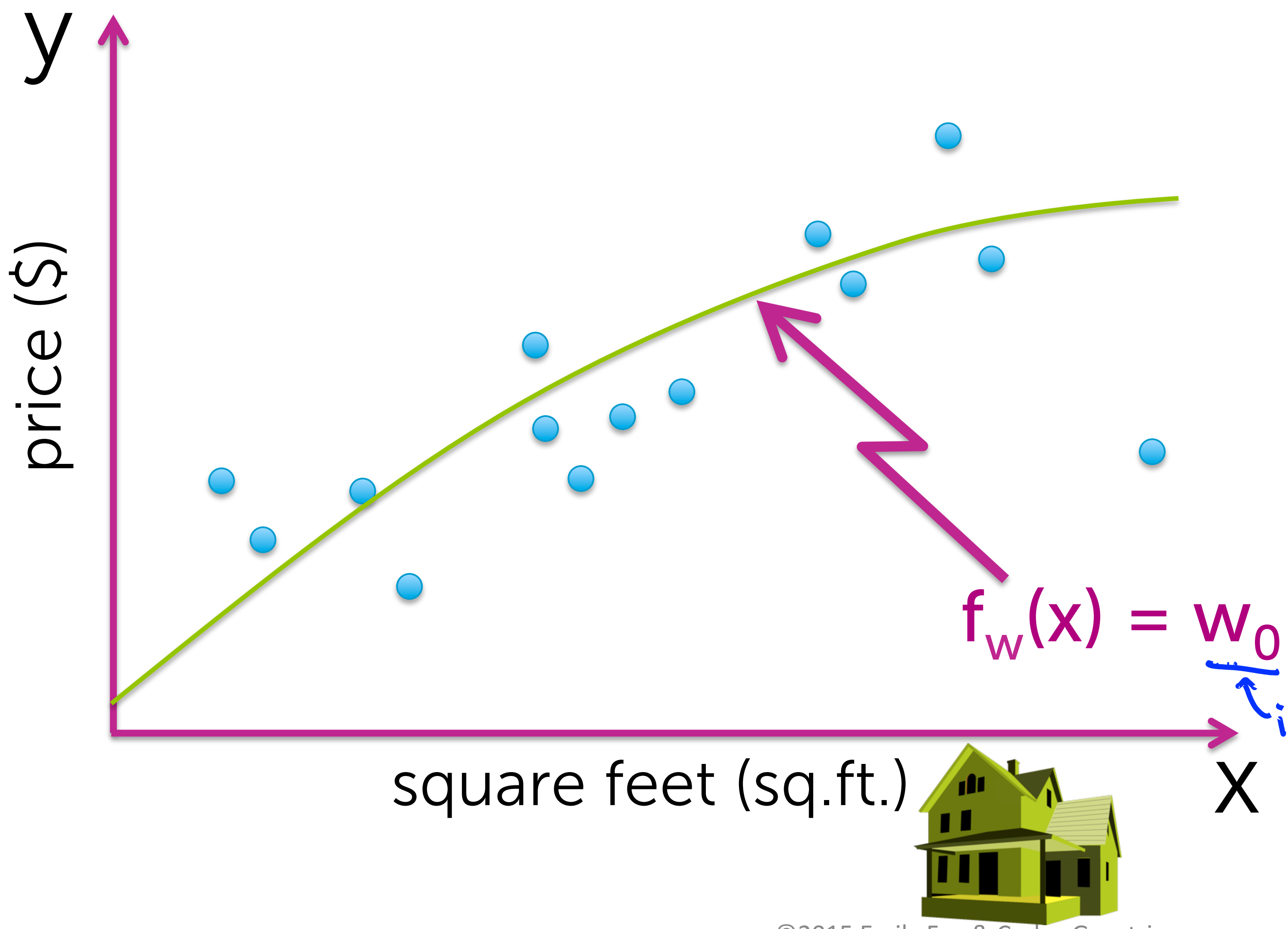
Fit data with a line or … ?



What about a quadratic function?

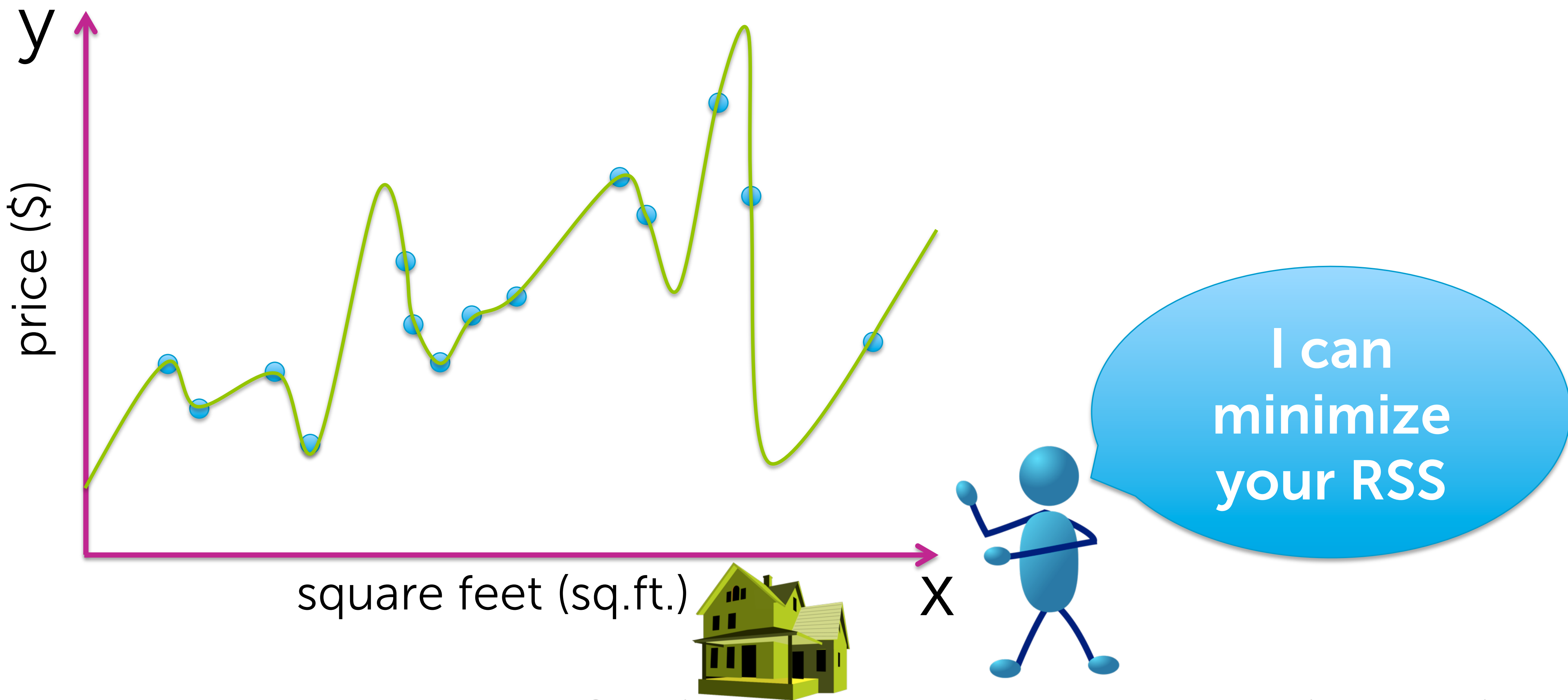


What about a quadratic function?

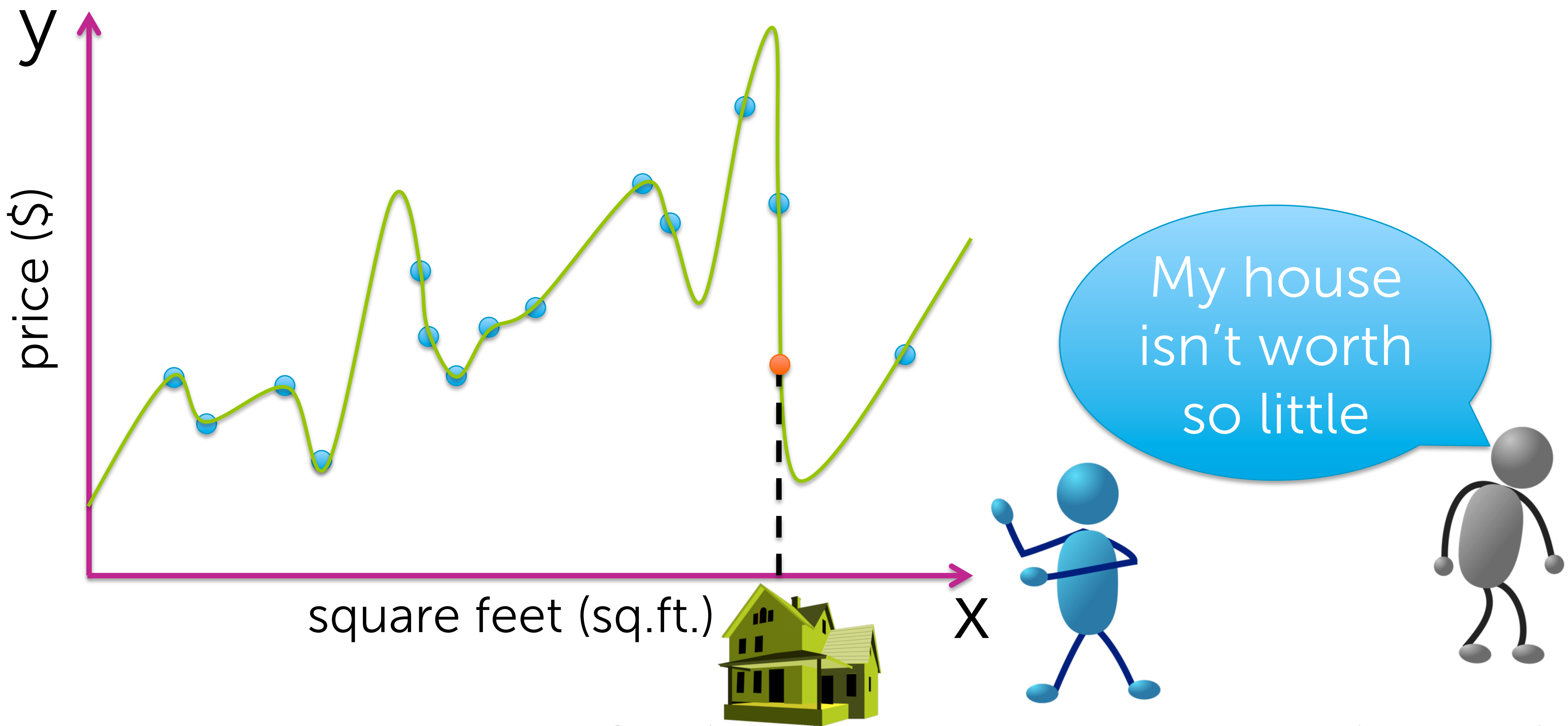


$$f_w(x) = \underbrace{w_0}_{\text{intercept}} + w_1 x + w_2 \overset{\text{just another feature}}{\boxed{x^2}}$$

Even higher order polynomial

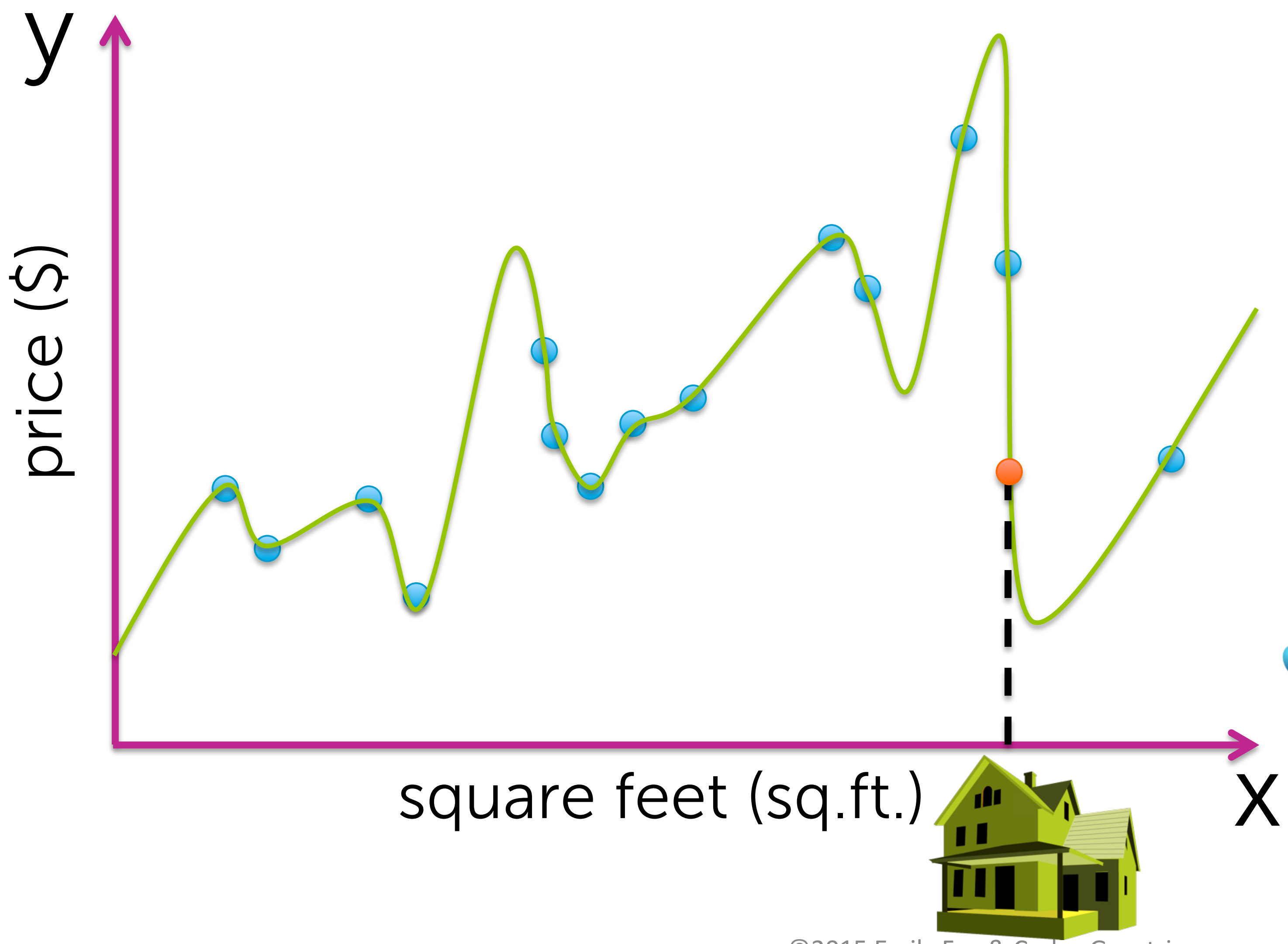


Do you believe this fit?



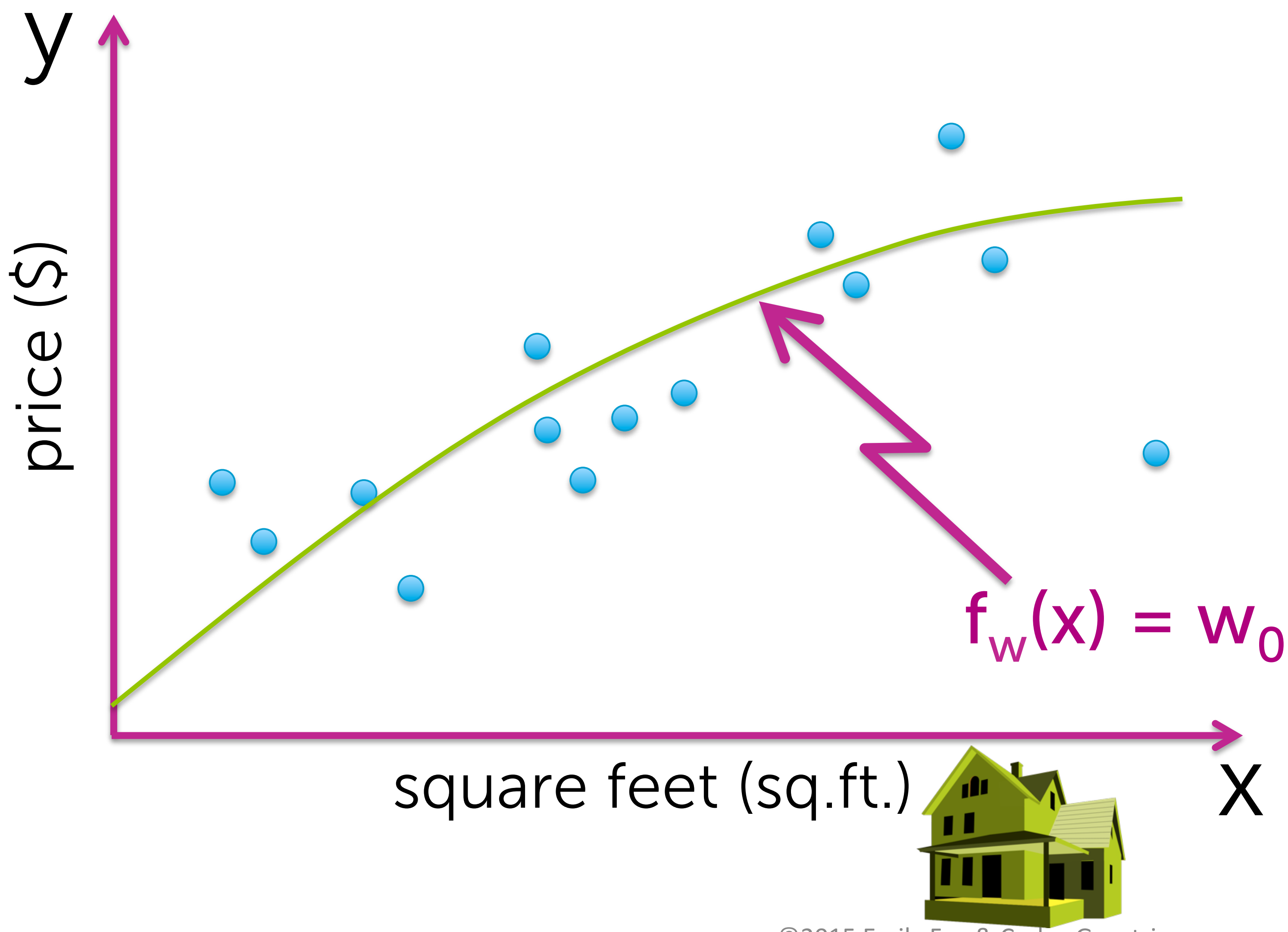
Evaluating overfitting via training/test split

Do you believe this fit?



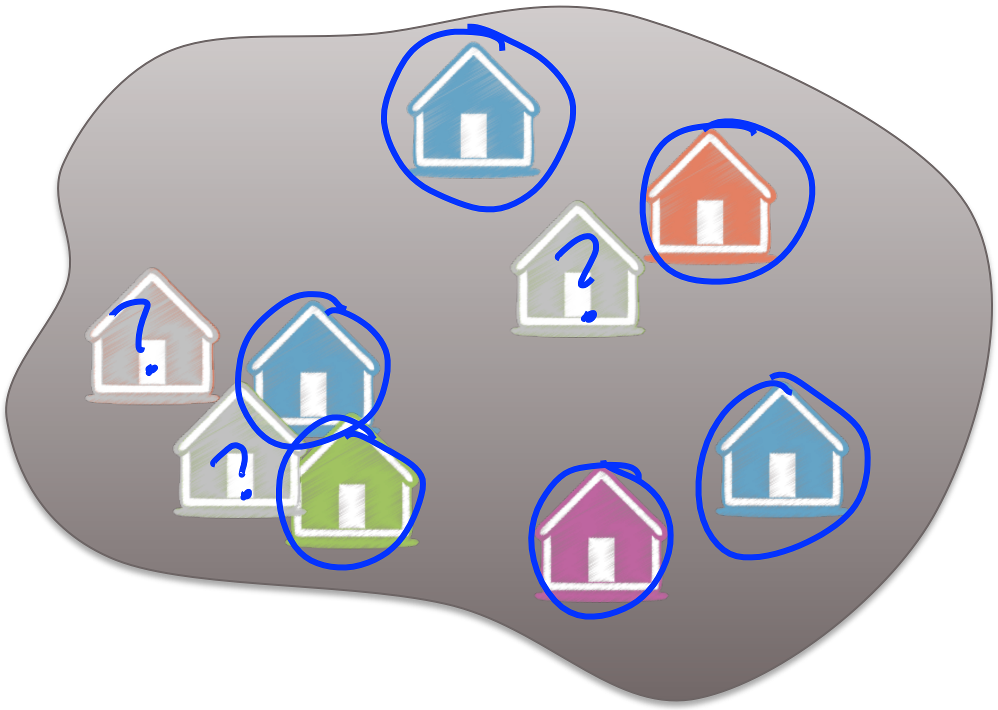
Minimizes RSS,
but bad predictions

What about a quadratic function?



$f_w(x) = w_0 + w_1 x + w_2 x^2$

How to choose model order/complexity



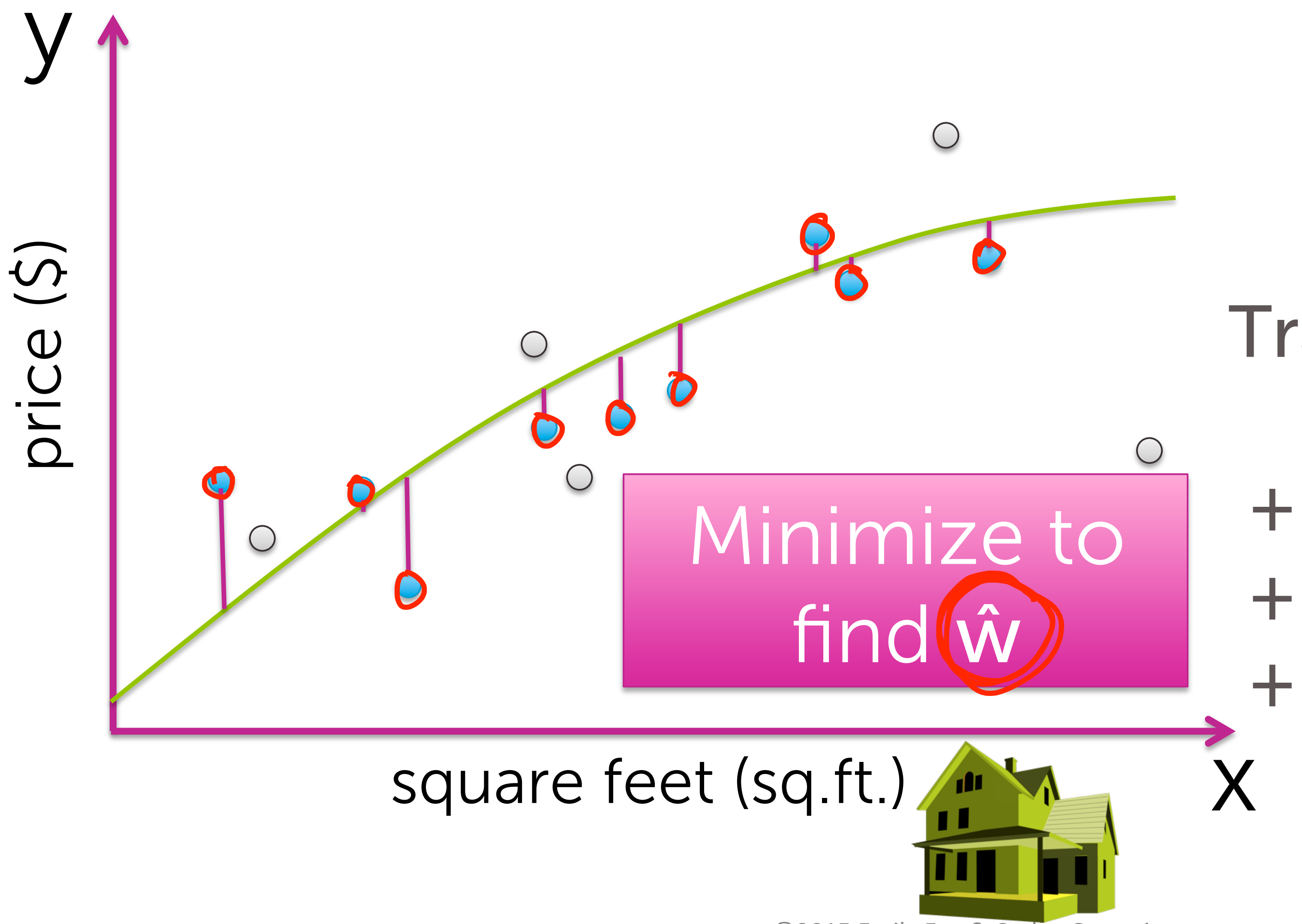
- Want good predictions, but can’t observe future
- **Simulate predictions**

1. Remove some houses
2. Fit model on remaining
3. Predict heldout houses

Training/test split

Terminology:
- training set
- test set

Training error

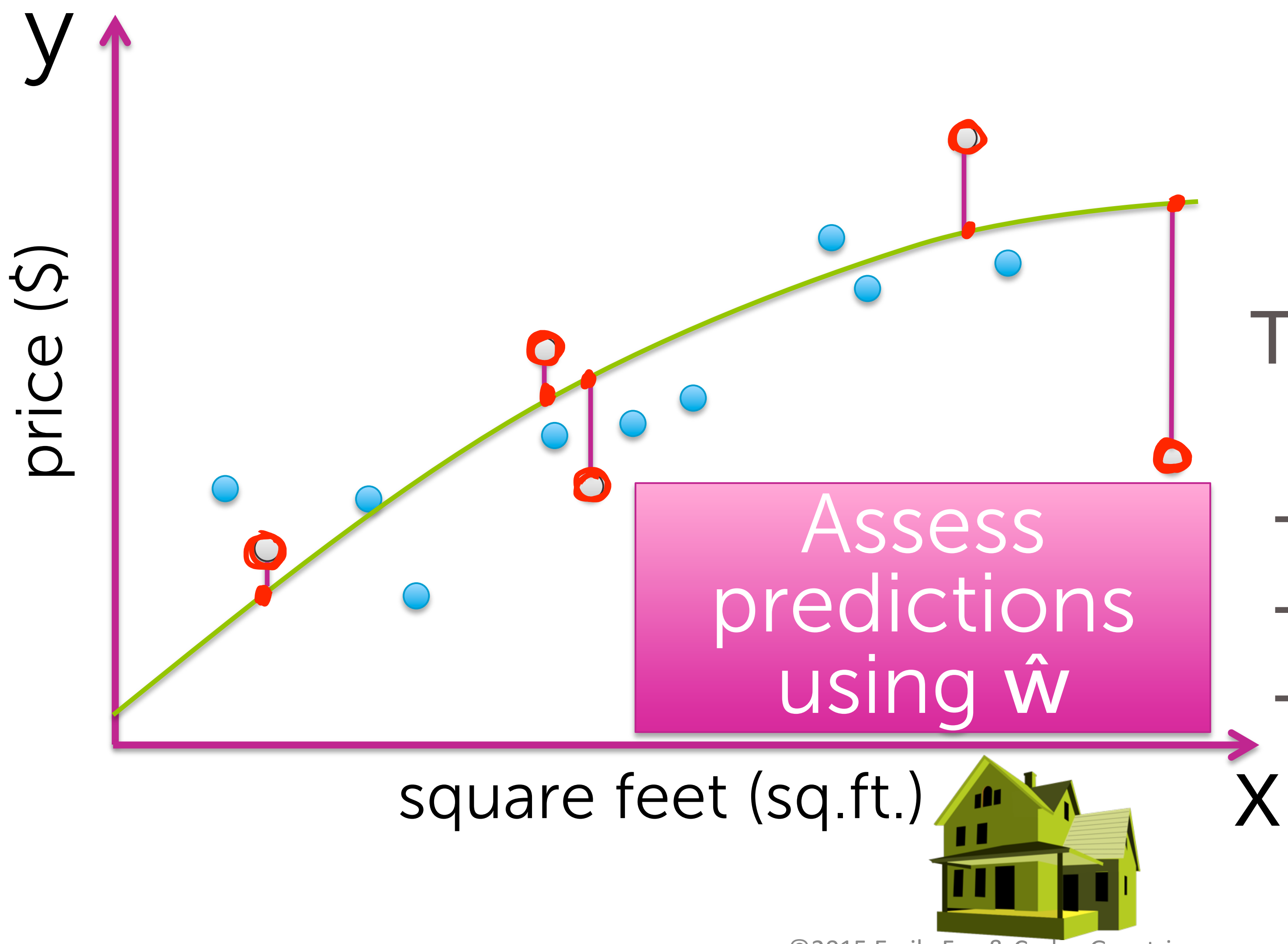


Training error ($\mathbf{w}$) =

$$(\$_{\text{train }1}-f_{\mathbf{w}}(\text{sq.ft.}_{\text{train }1}))^2$$
$$+\ (\$_{\text{train }2}-f_{\mathbf{w}}(\text{sq.ft.}_{\text{train }2}))^2$$
$$+\ (\$_{\text{train }3}-f_{\mathbf{w}}(\text{sq.ft.}_{\text{train }3}))^2$$

+ ... [include all training houses]

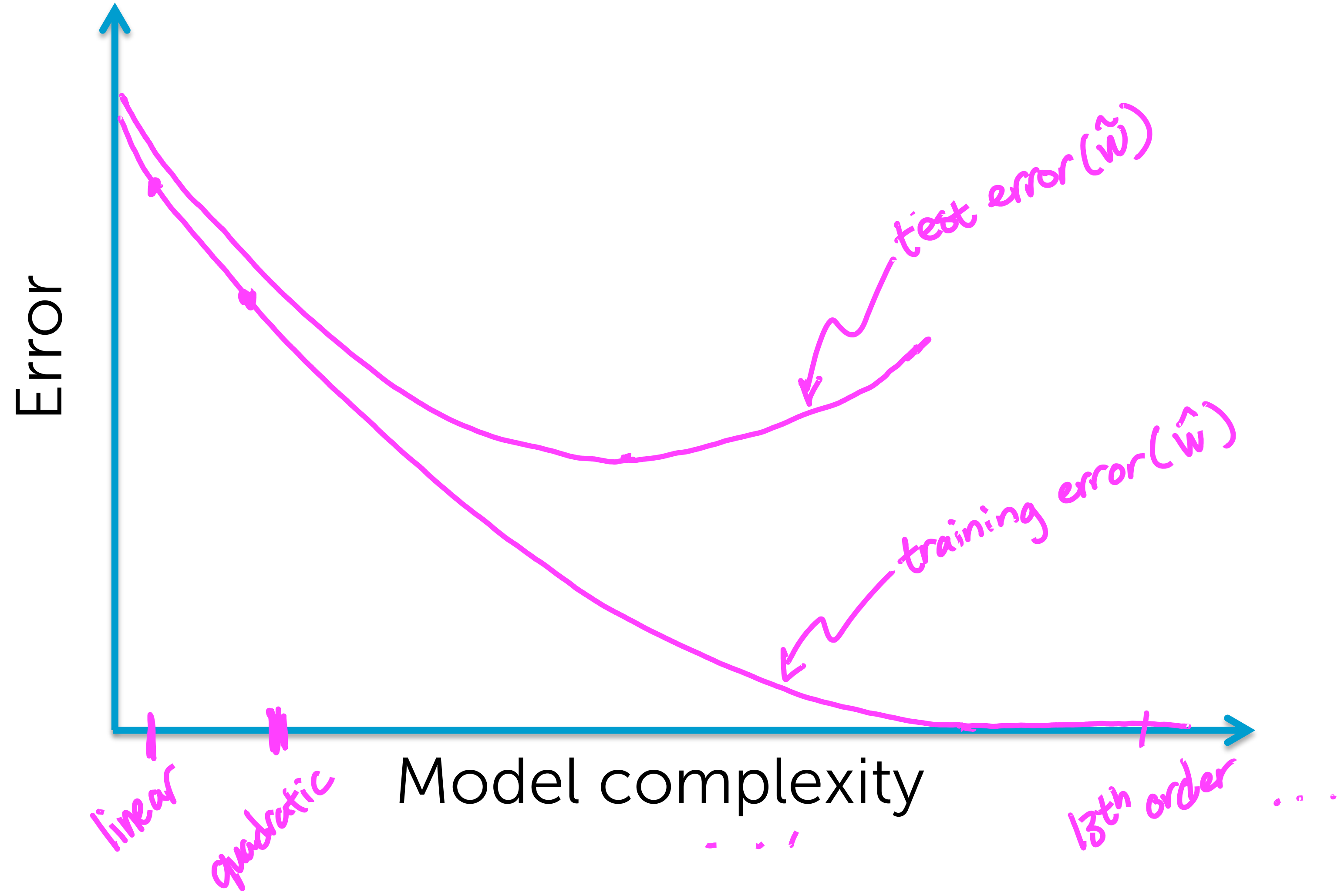
Test error



Assess predictions using ŵ

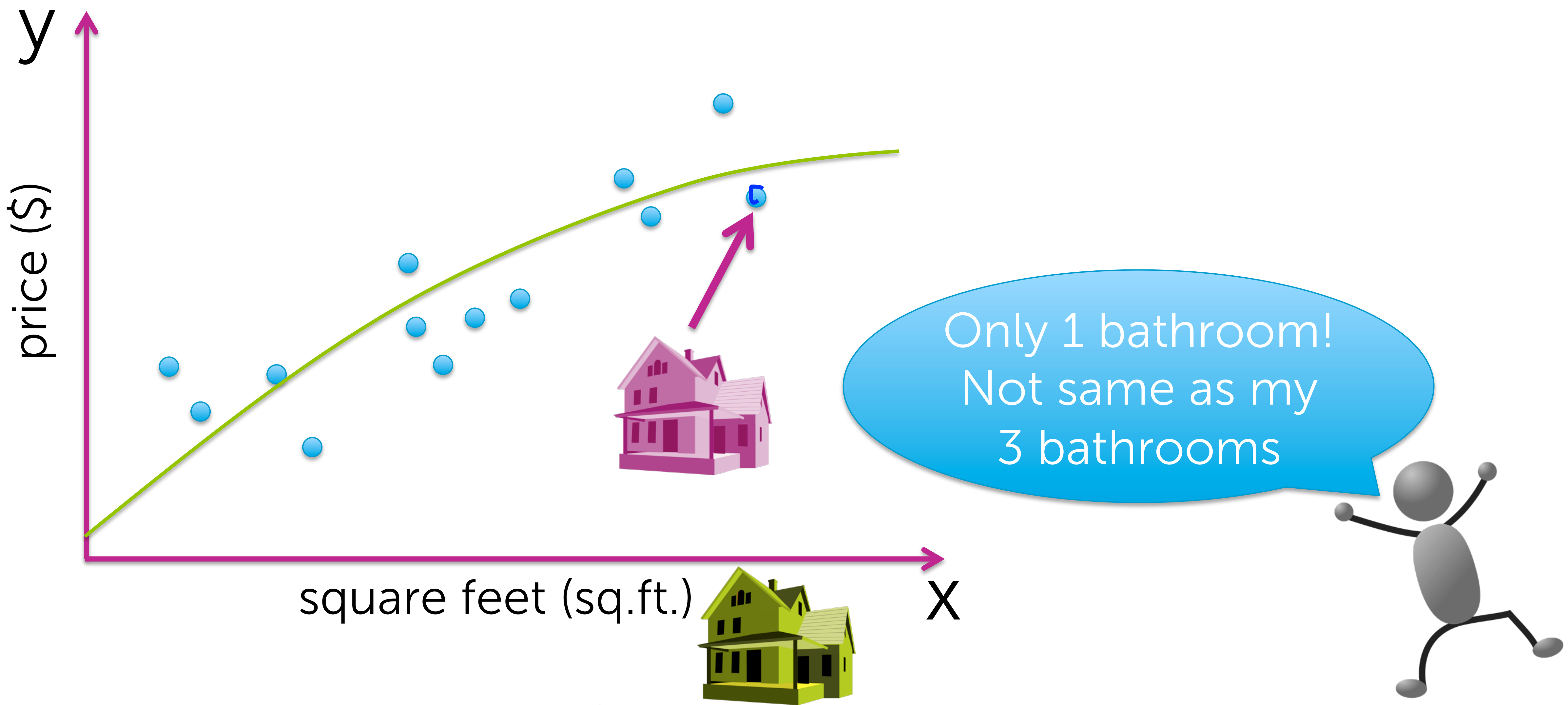
$$\text{Test error}(\hat{w}) = (\$_{\text{test }1} - f_{\hat{w}}(\text{sq.ft.}_{\text{test }1}))^2 + (\$_{\text{test }2} - f_{\hat{w}}(\text{sq.ft.}_{\text{test }2}))^2 + (\$_{\text{test }3} - f_{\hat{w}}(\text{sq.ft.}_{\text{test }3}))^2 + \ldots \text{ [include all test houses]}$$

Training/Test Curves



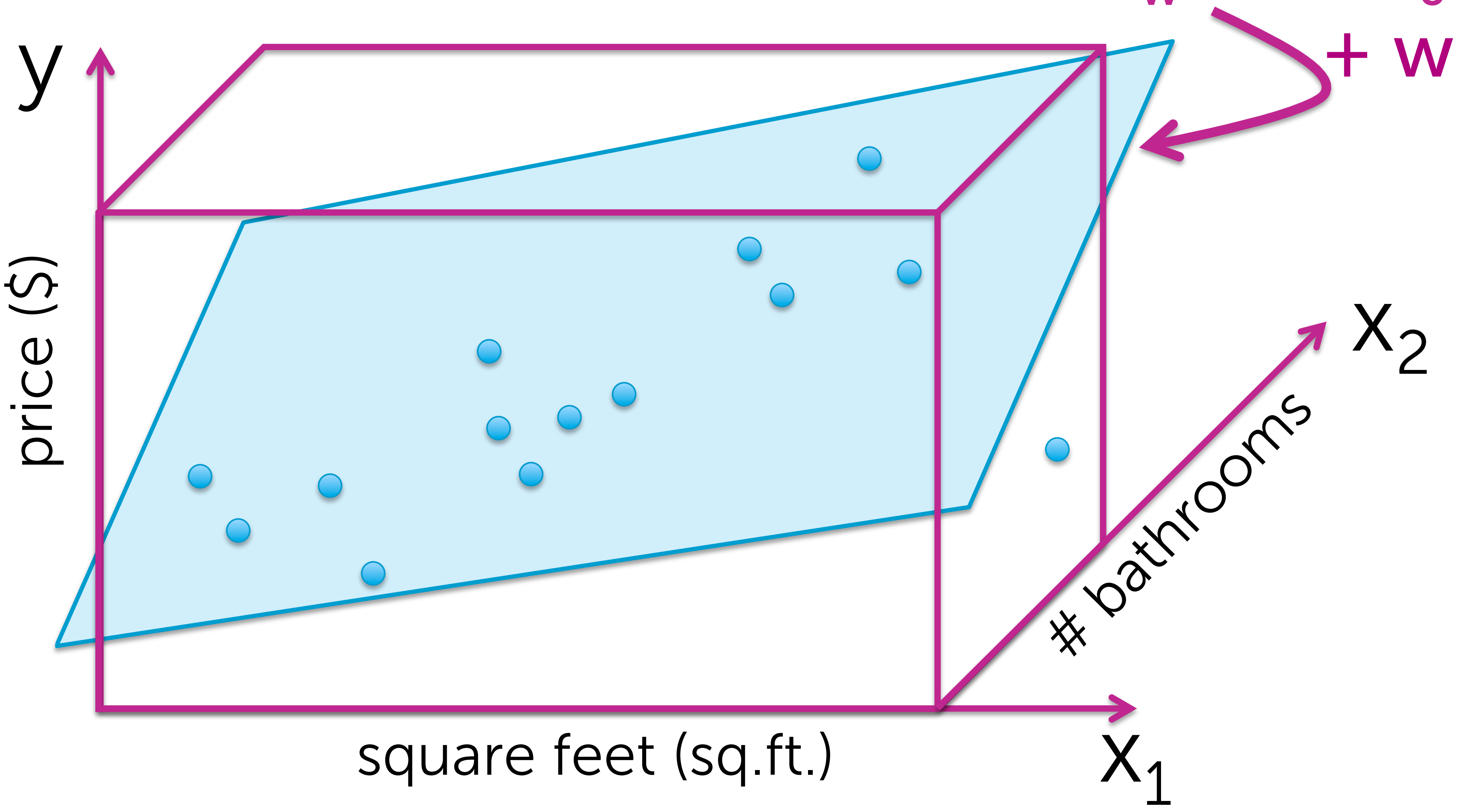
Adding other features

Predictions just based on house size



Add more features

$$f_w(x) = w_0 + w_1 \text{ sq.ft.} + w_2 \text{ \#bath}$$

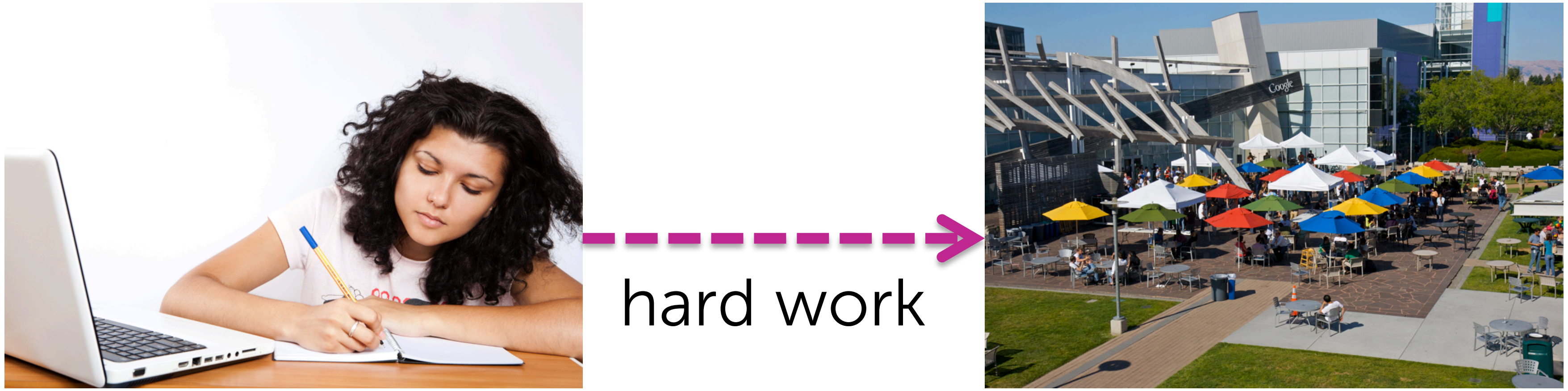


How many features to use?

- Possible choices:
  - Square feet
  - # bathrooms
  - # bedrooms
  - Lot size
  - Year built
  - …
- **See Regression Course!**

Other regression examples

Salary after ML specialization



hard work

- How much will your salary be? (**y** = $$)
- Depends on **x** = performance in courses, quality of capstone project, # of forum responses, …

Salary after ML specialization

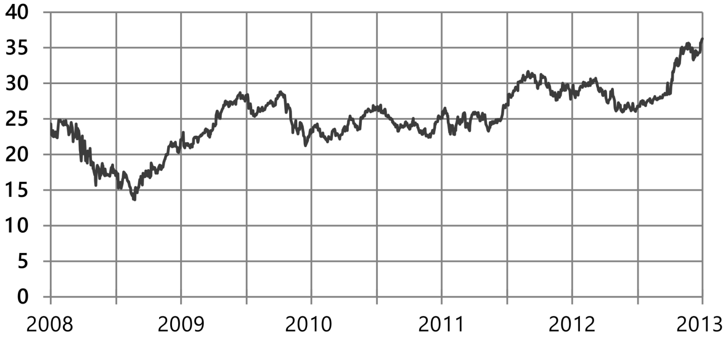
hard work

$$\hat{y} = \hat{w}_0 + \hat{w}_1 \, \text{performance} + \hat{w}_2 \, \text{capstone} + \hat{w}_3 \, \text{forum}$$

informed by other students who completed specialization

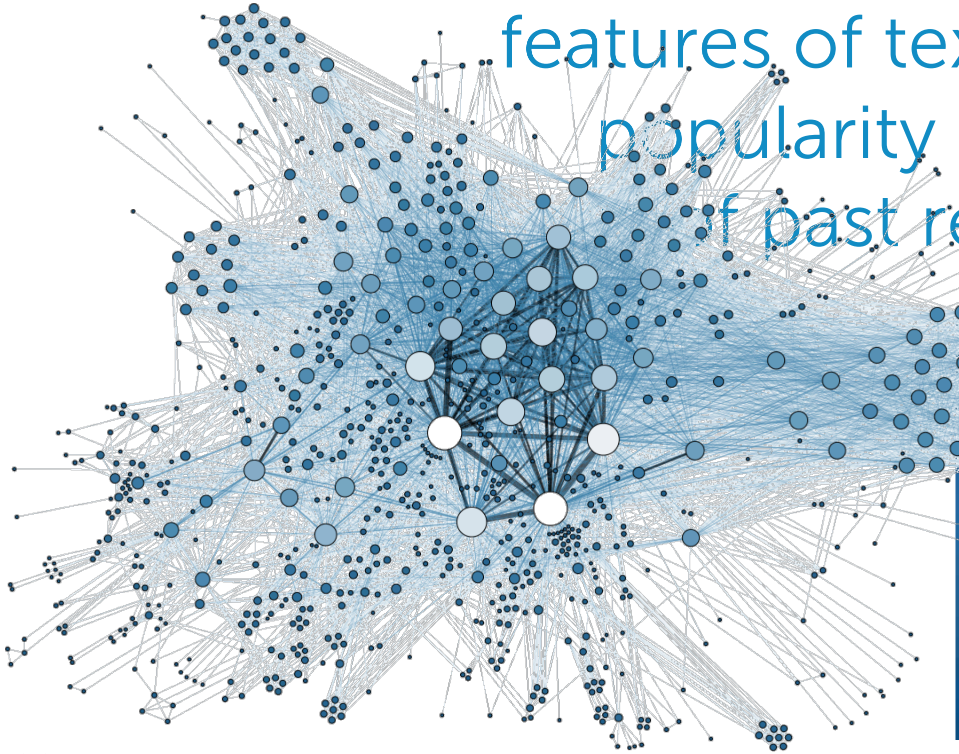
Stock prediction

- Predict the price of a stock
- Depends on
  - Recent history of stock price
  - News events
  - Related commodities



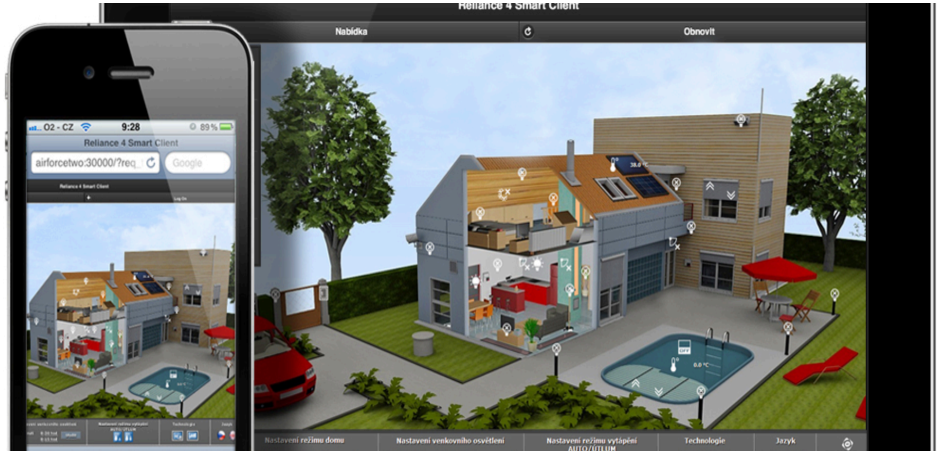
Tweet popularity

- How many people will retweet your tweet?
- Depends on # followers,
  # of followers of followers,
  features of text tweeted,
  popularity of hashtag,
  # of past retweets,...



Smart houses

- Smart houses have many distributed sensors
- What's the temperature at your desk? (no sensor)
  - Learn spatial function to predict temp
- Also depends on
  - Thermostat setting
  - Blinds open/closed or window tint
  - Vents
  - Temperature outside
  - Time of day



Summary for regression

What you can do now…

- Describe the input (features) and output (real-valued predictions) of a regression model
- Calculate a goodness-of-fit metric (e.g., RSS)
- Estimate model parameters by minimizing RSS (algorithms to come…)
- Exploit the estimated model to form predictions
- Perform a training/test split of the data
- Analyze performance of various regression models in terms of test error
- Use test error to avoid overfitting when selecting amongst candidate models
- Describe a regression model using multiple features
- Describe other applications where regression is useful